Math 10 Lecture Videos

Section 2.7:
Solving Linear Inequalities

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OBJECTIVE:

1. Graph the solutions of an inequality on a number line.
2. Use interval notation.
3. Understand the properties used to solve linear equations.
4. Solve linear inequalities.
5. Identify inequalities with no solution or true for all real numbers.
6. Solve a problem using linear inequalities.

Objective 1: Graph the solutions of an inequality on a number line.

Linear Inequality in One Variable

Any inequality in the form of $ax + b \leq c$

The inequality symbol may be

- $<$ less than
- $>$ greater than
- $\leq$ less than or equal to
- $\geq$ greater than or equal to

Objective 1: Graph the solutions of an inequality on a number line.

Graphing the Inequality

Let's consider this example:

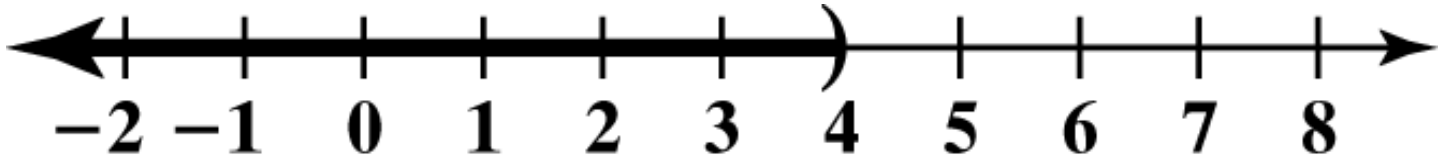
$$x < 4$$

- Infinitely many solutions to the inequality: all real numbers that are less than 3.
- Though we cannot list all the solutions, we can draw a number line that represents these solutions.

Objective 1: Graph the solutions of an inequality on a number line.

Graphing the Inequality

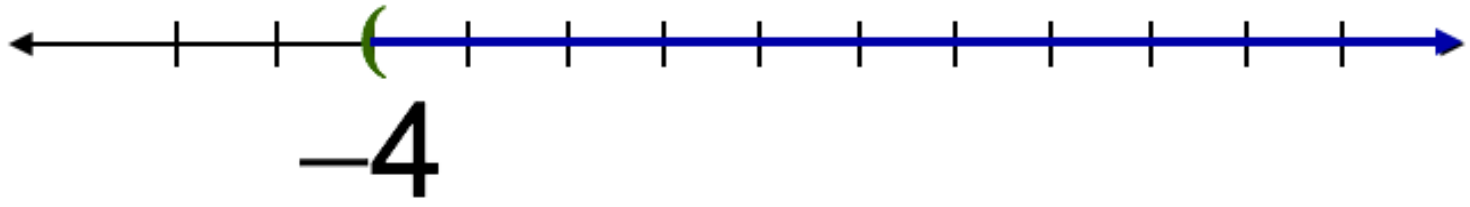
Let's consider this example: $x < 4$



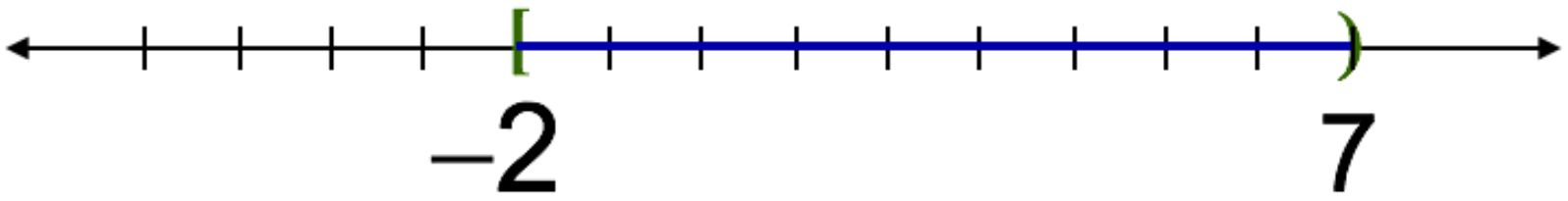
- Shade all points representing numbers that are solutions.
- Square brackets, [ ], indicate endpoints that are solutions
- Parentheses, ( ), indicate endpoints that are not solutions.
- Parentheses, ( ), are always used with ∞ and - ∞.

Objective 1: Graph the solutions of an inequality on a number line.

Example 1: x > -4



Example 2: -2 ≤ x < 7



Objective 2: Use interval notation

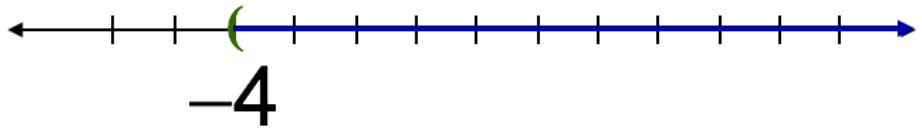
Let a and b be real numbers.

Inequality	Interval Notation	Set-Builder Notation	Graph
$x > a$	(a, ∞)	$\{x \mid x > a\}$	
$x \geq a$	$[a, \infty)$	$\{x \mid x \geq a\}$	
$x < b$	$(-\infty, b)$	$\{x \mid x < b\}$	
$x \leq b$	$(-\infty, b]$	$\{x \mid x \leq b\}$	

NOTE:
Interval Notation – a pair of numbers that represents the endpoints of the interval
Set-Builder Notation - shorthand used to write sets, often sets with an infinite number of elements.

Objective 2: Use interval notation

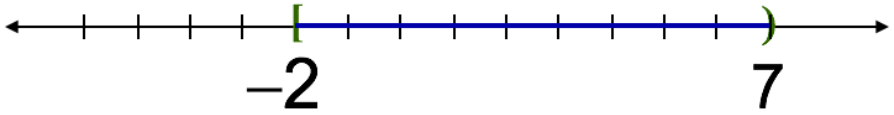
Example 1: $x > -4$



Interval Notation: $(-4, \infty)$

Set Builder Notation: $\{x \mid x > -4\}$

Example 2: $-2 \leq x < 7$



Interval Notation: $[-2, 7)$

Set Builder Notation: $\{x \mid -2 \leq x < 7\}$

Objective 3: Understand properties used to solve linear inequalities.

Properties of Inequalities *(continued)*		
Property	**The Property in Words**	**Example**
The Addition Property of Inequality If $a < b$, then $a + c < b + c$ If $a < b$, then $a - c < b - c$	If the same quantity is added to or subtracted from both sides of an inequality, the resulting inequality is equivalent to the original one.	$2x + 3 < 7$ Subtract 3: $2x + 3 - 3 < 7 - 3$ Simplify: $2x < 4$

Objective 3: Understand properties used to solve linear inequalities.

Properties of Inequalities *(continued)*

Property	The Property in Words	Example
The Positive Multiplication Property of Inequality If $a < b$, and c is positive, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.	If we multiply or divide both sides of an inequality by the same *positive* quantity, the resulting inequality is equivalent to the original one.	$2x < 4$ Divide by 2: $\frac{2x}{2} < \frac{4}{2}$ Simplify: $x < 2$

Objective 3: Understand properties used to solve linear inequalities.

Properties of Inequalities *(continued)*

Property	The Property in Words	Example
The Negative Multiplication Property of Inequality If $a < b$, and c is negative, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.	If we multiply or divide both sides of an inequality by the same *negative* quantity and reverse the direction of the inequality symbol, the resulting inequality is equivalent to the original one.	$-4x < 20$ Divide by -4 and reverse the direction of the inequality: $\frac{-4x}{-4} > \frac{20}{-4}$ Simplify: $X > -5$

Objective 3: Understand properties used to solve linear inequalities.

True or False:

When we add (or subtract) a negative number to (or from) both sides of an inequality, the direction of the inequality symbol is reversed.

False; This rule applies to multiplication and division.

Objective 4: Solve linear inequalities

1. Simplify the algebraic expression on each side.
2. Use the addition property of inequality to collect all the variable terms on one side and all the constant terms on the other side.
3. Use the multiplication property of inequality to isolate the variable and solve. *Reverse the sense of the inequality when multiplying or dividing both sides by a negative number.*
4. Express the solution set in set-builder or interval notation and graph the solution set on a number line.

Objective 4: Solve linear inequalities

Solve: $2(x + 3) \leq 4x + 12$

$$2(x+3) \leq 4x+12$$

$$2x+6 \leq 4x+12 \quad \text{Distribute}$$

$$2x+6-4x \leq 4x+12-4x \quad \text{Subtract } 4x \text{ from both sides}$$

$$-2x+6 \leq 12 \quad \text{Subtract 6 from both sides}$$

$$-2x \leq 6 \quad \text{Divide both sides by } -2$$

$$x \geq -3$$

Interval Notation: $[-3, \infty)$

Set-Builder Notation: $\{x \mid x \geq -3\}$

Objective 4: Solve linear inequalities

Solve: **-6x < 18**

$$-6x < 18$$

$$\frac{-6x}{-6} > \frac{18}{-6}$$

$$x > -3$$

Interval Notation: (-3, ∞)

Set-Builder Notation: {x | x > −3}

Objective 4: Solve linear inequalities

Solve: $\frac{1}{4}x < 2$

$$\frac{1}{4}x < 2$$

$$4 \cdot \frac{1}{4}x < 4 \cdot 2$$

$$x < 8$$

Interval Notation: $(-\infty, 8)$

Set-Builder Notation: $\{x \mid x < 8\}$

Objective 5: Identify inequalities with no solution or true for all real numbers

- Some inequalities have either **no solution** or have **infinitely many solutions**.
- When you attempt to solve such inequalities, all the variables are eliminated and you either get a sentence that is true or false.
    - If you are left with a true sentence, the original inequality is an identity and is true for all x.
    - If you are left with a false sentence, the original inequality has no solution.

Objective 5: Identify inequalities with no solution or true for all real numbers

- If you are left with a true sentence, the original inequality is an identity and is true for all x.

Example: $3(x + 1) \geq 2x + 1 + x$

$$3(x+1) \geq 2x+1+x$$
$$3x+3 \geq 3x+1$$
$$3x-3x+3 \geq 3x-3x+1$$
$$3 \geq 1,$$

$(-\infty, \infty)$

$\{x \mid x \text{ is a real number}\}$.

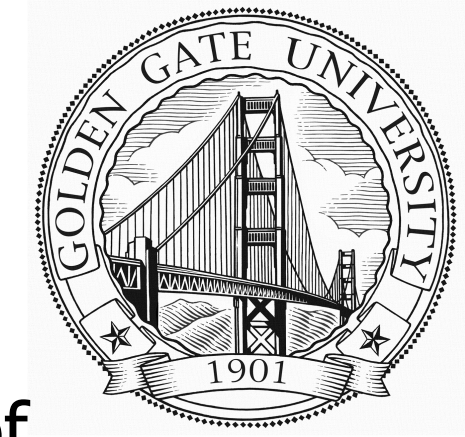
Objective 5: Identify inequalities with no solution or true for all real numbers

- If you are left with a false sentence, the original inequality has no solution.

Example: $4(x + 2) > 4x + 15$

$$4(x+2) > 4x+15$$
$$4x+8 > 4x+15$$
$$4x-4x+8 > 4x-4x+15$$
$$8 > 15, \text{ FALSE: There is no solution or } \{\ \}.$$

Objective 5: Solve a problem using linear inequalities



To earn a B in a course, you must have a final average of at least 80%. On the first three examinations, you have grades of 82%, 74%, and 78%. If the final examination counts as two grades, what must you get on the final to earn a B in the course?

Let x = your grade on the final examination.

The average of examinations must be greater than or equal to 80.

Objective 5: Solve a problem using linear inequalities

To earn a B in a course, you must have a final average of at least 80%. On the first three examinations, you have grades of 82%, 74%, and 78%. If the final examination counts as two grades, what must you get on the final to earn a B in the course?

$$\frac{82+74+78+x+x}{5} \geq 80$$

$$\frac{234+2x}{5} \geq 80$$

$$5\left(\frac{234+2x}{5}\right) \geq 5.80$$

$$234+2x \geq 400$$

$$234-234+2x \geq 400-234$$

$$2x \geq 166$$

$$x \geq 83$$

To earn a B you must get at least an 83% on the final examination.

OBJECTIVE:

1. Graph the solutions of an inequality on a number line. ✓
2. Use interval notation. ✓
3. Understand the properties used to solve linear equations.
4. Solve linear inequalities. ✓
5. Identify inequalities with no solution or true for all real numbers. ✓
6. Solve a problem using linear inequalities. ✓